

Exercise 8

1. Prove that

(a) For any f (convex or not), x_0 is a minimizer of f if and only if $0 \in \partial f(x_0)$;

(b) Let $f : X \rightarrow \mathbb{R}$ be convex and differentiable and X be a convex set.

Then $0 \in \partial f(x_0)$ if and only if

$$\langle \nabla f(x_0), x - x_0 \rangle \geq 0, \forall x \in X.$$

2. Let f be a convex function. Prove that x_0 is a local minimum of f , then x_0 is a global minimum of f .